Dependence of $|V_{ub}/V_{cb}|$ on Fermi momentum p_F in ACCMM model

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Abstract

The Gaussian width of Fermi momentum, p_F, is the most important parameter of the ACCMM model, and its value is essential in the determination of $|V_{ub}/V_{cb}|$ because the experimental analysis is allowed only at the end-point region of inclusive semileptonic B-decay spectrum. We extract the value of $|V_{ub}/V_{cb}|$ as a function of p_F. We also calculate the parameter p_F in the relativistic quark model using the variational method, and obtain $p_F = 0.54$ GeV which is much larger than the commonly used value, ~ 0.3 GeV, in experimental analyses. When we use $p_F = 0.5$ GeV instead of 0.3 GeV, the value of $|V_{ub}/V_{cb}|$ from ACCMM model is increased by a factor 1.81, and can give a good agreement with Isgur *et al.* model.

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1. Introduction

In the minimal standard model CP violation is possible through the CKM mixing matrix of three families, and it is important to know whether the element V_{ub} is non-zero or not accurately. Its knowledge is also necessary to check whether the unitarity triangle is closed or not [1]. However, its experimental value is very poorly known presently and its better experimental information is urgently required. At present, the only experimental method to measure V_{ub} is through the end-point lepton energy spectrum of the inclusive B-meson semileptonic decays, *e.g.* CLEO [2] and ARGUS [3], and their data indicate that V_{ub} is non-zero. Recently it has also been suggested that the measurements of hadronic invariant mass spectrum [4] as well as hadronic energy spectrum [5] in the inclusive $B \rightarrow X_{c(u)}l\nu$ decays can be useful in extracting $|V_{ub}|$ with better theoretical understandings. In future asymmetric B factories with vertex detector, they will offer alternative ways to select $b \rightarrow u$ transitions that are much more efficient than selecting the upper end of the lepton energy spectrum.

The simplest model for the semileptonic B-decay is the spectator model which considers the decaying b-quark in the B-meson as a free particle. The spectator model is usually used with the inclusion of perturbative QCD radiative corrections [6]. Then the decay width of the process $B \rightarrow X_q l\nu$ is given by

$$\Gamma_B(B \rightarrow X_q l\nu) \simeq \Gamma_b(b \rightarrow ql\nu) = |V_{bq}|^2 (\frac{G_F^2 m_b^5}{192\pi^3}) f(\frac{m_q}{m_b})[1 - \frac{2}{3}\frac{\alpha_s}{\pi}g(\frac{m_q}{m_b})], \quad (1)$$

where m_q is the mass of the final q-quark decayed from b-quark. As can be seen, the decay width of the spectator model depends on m_b^5, therefore small difference of m_b would change the decay width significantly.

Altarelli *et al.* [7] proposed for the inclusive B-meson semileptonic decays their ACCMM model, which incorporates the bound state effect by treating the b-quark as a virtual state particle, thus giving momentum dependence to the b-quark mass.

The virtual state b-quark mass W is given by

$$W^2(\mathbf{p}) = m_B^2 + m_{sp}^2 - 2m_B\sqrt{\mathbf{p}^2 + m_{sp}^2} \qquad (2)$$

in the B-meson rest frame, where m_{sp} is the spectator quark mass, m_B is the B-meson mass, and $\mathbf{p}$ is the momentum of the b-quark inside B-meson.

For the momentum distribution of the virtual b-quark, Altarelli $et.$ $al.$ considered the Fermi motion inside the B-meson with the Gaussian momentum distribution

$$\phi(\mathbf{p}) = \frac{4}{\sqrt{\pi}p_F^3}e^{-\mathbf{p}^2/p_F^2}, \qquad (3)$$

where the Gaussian width, p_F, is treated as a free parameter. Then the lepton energy spectrum of the B-meson decay is given by

$$\frac{d\Gamma_B}{dE_l}(p_F, m_{sp}, m_q, m_B) = \int_0^{p_{max}} dp\ p^2 \phi(\mathbf{p})\ \frac{d\Gamma_b}{dE_l}(m_b = W, m_q), \qquad (4)$$

where p_{max} is the maximum kinematically allowed value of $p = |\mathbf{p}|$. The ACCMM model, therefore, introduces a new parameter p_F for the Gaussian momentum distribution of the b-quark inside B-meson instead of the b-quark mass of the spectator model. In this way the ACCMM model incorporates the bound state effects and reduces the strong dependence on b-quark mass in the decay width of the spectator model.

The Fermi momentum p_F is the most essential parameter of the ACCMM model as we see in the above. However, the experimental determination of its value from the lepton energy spectrum has been very ambiguous, because various parameters of ACCMM model, such as p_F, m_q and m_{sp}, are fitted all together from the limited region of end-point lepton energy spectrum, and because the perturbative QCD corrections are very sensitive in the end-point region of the spectrum. Recently, ARGUS [8] extracted the lepton energy spectrum of $B \to X_c l\nu$ for the whole region of electron energy, but with much larger uncertainties. We argue that the value $p_F \sim 0.3$ GeV, which has been commonly used in experimental analyses, has no

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theoretical or experimental clear justification, even though there has been recently an assertion that the prediction of heavy quark effective theory approach [9], far from the end-point region, gives approximately equal shape to the ACCMM model with $p_F \sim 0.3$ GeV. Therefore, it is stongly recommended to determine the value of p_F more reliably and independently, when we think of the importance of its role in experimental analyses. It is particularly important in the determination of the value of $|V_{ub}/V_{cb}|$, as we explain in section 2. A better determination of p_F is also interesting theoretically since it has its own physical correspondence related to the Fermi motion inside B-meson. In this context we calculate theoretically the value of p_F in the relativistic quark model using quantum mechanical variational method in section 3. And we obtain $p_F = 0.54$ GeV which is much larger than 0.3 GeV. Section 4 contains the conclusion.

2. Dependence of $|V_{ub}/V_{cb}|$ on the Fermi momentum parameter p_F

The ACCMM model provides an inclusive lepton energy spectrum of the B-meson semileptonic decay to obtain the value of $|V_{ub}/V_{cb}|$. The leptonic energy spectrum is useful in separating $b \to u$ transitions from $b \to c$, since the end-point region of the spectrum is completely composed of $b \to u$ decays. In applying this method one integrates (4) in the range 2.3 GeV $< E_l <$ 2.6 GeV at the B-meson rest frame, where only $b \to u$ transitions exist [10]. So we theoretically calculate

$$\widetilde{\Gamma}(p_F) \equiv \int_{2.3}^{2.6} dE_l \frac{d\Gamma_B}{dE_l}(p_F, m_{sp}, m_q, m_B) \ . \tag{5}$$

In (5) we specified only p_F dependence explicitly in the left-hand side, and $d\Gamma_B/dE_l$ in the right-hand side is from (4). Then one compares the theoretically calculated $\widetilde{\Gamma}(p_F)$ with the experimentally measured width $\widetilde{\Gamma}_{exp}$ in the region 2.3 GeV $< E_l <$ 2.6 GeV, to extract the value of $|V_{ub}|$ from the relation

$$\widetilde{\Gamma}_{exp} = |V_{ub}|^2 \times \widetilde{\Gamma}(p_F) \ . \tag{6}$$

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In the real experimental situation [2, 3, 8, 10], the only measured quantity is the number of events in this region of high E_l compared to the total semileptonic events number, *i.e.* the branching-fraction $\widetilde{\Gamma}_{exp}/\widetilde{\Gamma}_{s.l.}^{total}$. Since the value $\widetilde{\Gamma}_{s.l.}^{total}$ is propotional to $|V_{cb}|^2$, only the combination $|V_{ub}/V_{cb}|^2$ is extracted.

We now consider the possible dependence of $|V_{ub}/V_{cb}|^2$ as a function of the parameter p_F from the following relation

$$\frac{\widetilde{\Gamma}_{exp}}{\widetilde{\Gamma}_{s.l.}^{total}} \quad \propto \quad \left|\frac{V_{ub}}{V_{cb}}\right|^2_{p_F=p_F} \times \widetilde{\Gamma}(p_F) = \left|\frac{V_{ub}}{V_{cb}}\right|^2_{p_F=0.3} \times \widetilde{\Gamma}(p_F=0.3) \ , \qquad (7)$$

where $|V_{ub}/V_{cb}|^2_{p_F=p_F}$ is determined with an arbitrary value of the Fermi momentum parameter p_F. In the right-hand side we used $p_F=0.3$ GeV because this value is commonly used in the experimental determination of $|V_{ub}/V_{cb}|$. Then one can get a relation

$$\left|\frac{V_{ub}}{V_{cb}}\right|^2_{p_F=p_F} = \left|\frac{V_{ub}}{V_{cb}}\right|^2_{p_F=0.3} \times \frac{\widetilde{\Gamma}(0.3)}{\widetilde{\Gamma}(p_F)} \ . \qquad (8)$$

In section 3, we obtain $p_F = 0.54$ GeV using the variational method in the relativistic quark model. If we use $p_F = 0.5$ GeV, instead of $p_F = 0.3$ GeV, in the experimental analysis of the end-point region of lepton energy spectrum, the value of $|V_{ub}/V_{cb}|$ becomes significantly changed. We numerically calculated theoretical ratio $\widetilde{\Gamma}(0.3)/\widetilde{\Gamma}(0.5)$ by using (4) and (5) with $m_{sp} = 0.15$ GeV, $m_q = 0.15$ GeV, which are the values commonly used by experimentalists, and $m_B = 5.28$ GeV, to get its value as 1.81, which finally gives

$$\left|\frac{V_{ub}}{V_{cb}}\right|^2_{p_F=0.5} = \left|\frac{V_{ub}}{V_{cb}}\right|^2_{p_F=0.3} \times 1.81 \ . \qquad (9)$$

Previously the CLEO [10] analyzed with $p_F = 0.3$ GeV the end-point lepton energy spectrum to get

$$10^2 \times |V_{ub}/V_{cb}|^2 = 0.57 \pm 0.11 \ \ (\text{ACCMM [7]})$$
$$= 1.02 \pm 0.20 \ \ (\text{Isgur et.al. [11]}) \ . \qquad (10)$$

As can be seen, those values are in large disagreement. However, if we use $p_F = 0.5$ GeV, the result of the ACCMM model becomes 1.03, and these two models are

in a good agreement for the value of $|V_{ub}/V_{cb}|$. Finally we show the values of $|V_{ub}(p_F)/V_{ub}(p_F = 0.3)|$ as a function of p_F in Fig. 1.

3. Calculation of p_F in the relativistic quark model

We consider the Gaussian probability distribution function $\phi(\mathbf{p})$ in (3) as the absolute square of the momentum space wave function $\chi(\mathbf{p})$ of the bound state B-meson, that is,

$$\phi(\mathbf{p}) = 4\pi|\chi(\mathbf{p})|^2, \quad \chi(\mathbf{p}) = \frac{1}{(\sqrt{\pi}p_F)^{3/2}}e^{-\mathbf{p}^2/2p_F^2}. \tag{11}$$

The Fourier transform of $\chi(\mathbf{p})$ gives the coordinate space wave function $\psi(\mathbf{r})$, which is also Gaussian,

$$\psi(\mathbf{r}) = (\frac{p_F}{\sqrt{\pi}})^{3/2}e^{-r^2p_F^2/2}. \tag{12}$$

Then we can approach the determination of p_F in the framework of quantum mechanics. For the B-meson system we treat the b-quark non-relativistically, but the u- or d-quark relativistically with the Hamiltonian

$$H = M + \frac{\mathbf{p}^2}{2M} + \sqrt{\mathbf{p}^2 + m^2} + V(r), \tag{13}$$

where $M = m_b$ is the b-quark mass and $m = m_{sp}$ is the u- or d-quark mass. We apply the variational method to the Hamiltonian (13) with the trial wave function

$$\psi(\mathbf{r}) = (\frac{\mu}{\sqrt{\pi}})^{3/2}e^{-\mu^2r^2/2}, \tag{14}$$

where μ is the variational parameter. The ground state is given by minimizing the expectation value of H,

$$\langle H \rangle = \langle\psi|H|\psi\rangle = E(\mu), \quad \frac{d}{d\mu}E(\mu) = 0 \text{ at } \mu = \bar{\mu}, \tag{15}$$

and then $\bar{\mu} = p_F$ and $\bar{E} \equiv E(\bar{\mu})$ approximates m_B. The value of μ or p_F corresponds to the measure of the radius of the two body bound state as can be seen from $\langle r \rangle = \frac{2}{\sqrt{\pi}}\frac{1}{\mu}$ and $\langle r^2 \rangle^{\frac{1}{2}} = \frac{3}{2}\frac{1}{\mu}$.

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In (13) we take the Cornell potential which is composed of the Coulomb and linear potentials,

$$V(r) = -\frac{\alpha_c}{r} + Kr. \tag{16}$$

For the values of the parameters α_c ($\equiv \frac{4}{3}\alpha_s$), K, and the b-quark mass m_b, we use the values given by Hagiwara *et al.* [12],

$$\alpha_c = 0.47 \ (\alpha_s = 0.35), \quad K = 0.19 \ GeV^2, \quad m_b = 4.75 \ GeV, \tag{17}$$

which have been determined by the best fit of all the known $(c\bar{c})$ and $(b\bar{b})$ bound states. For comparison we will also consider $\alpha_c = 0.32$ ($\alpha_s = 0.24$), which corresponds to $\alpha_s(Q^2 = m_B^2)$.

Before applying our variational method with the Gaussian trial wave function to the B-meson system, let us check the method by considering the $\Upsilon(b\bar{b})$ system. The Hamiltonian of the $\Upsilon(b\bar{b})$ system can be approximated by the non-relativistic Hamiltonian

$$H \simeq 2m_b + \frac{\mathbf{p}^2}{m_b} + V(r). \tag{18}$$

With the parameters in (17) (or with $\alpha_c = 0.32$), our variational method with the Gaussian trial wave function (14) gives $p_F = \bar{\mu} = 1.1$ GeV and $\bar{E} = E(\bar{\mu}) = 9.49$ GeV. Here $p_F = 1.1$ GeV corresponds to the radius $R(\Upsilon) = 0.2$ fm, and $\bar{E}(\Upsilon) = 9.49$ GeV is within 0.3 % error compared with the experimental value $E_{\exp} = m_\Upsilon = 9.46$ GeV. Therefore, the variational method with the non-relativistic Hamiltonian (18) gives fairly accurate results for the Υ ground state.

However, since the u- or d-quark in the B-meson is very light, the non-relativistic description can not be applied to the B-meson system. For example, when we apply the variational method with the non-relativistic Hamiltonian to the B-meson, we get the results

$$p_F = 0.29 \ GeV, \quad \bar{E} = 5.92 \ GeV \quad \text{for} \quad \alpha_s = 0.35, \tag{19}$$

$$p_F = 0.29 \ GeV, \quad \bar{E} = 5.97 \ GeV \quad \text{for} \quad \alpha_s = 0.24. \tag{20}$$

The above masses $\bar{E}$ are much larger compared to the experimental value $m_B = 5.28$ GeV, and moreover the expectation values of the higher terms in the non-relativistic perturbative expansion are bigger than those of the lower terms. Therefore, we can not apply the variational method with the non-relativistic Hamiltonian to the B-meson system.

Let us come back to our Hamiltonian (13) of the B-meson system. In our variational method the trial wave function is Gaussian both in the coordinate space and in the momentum space, so the expectation value of H can be calculated in either space from $\langle H \rangle = \langle \psi(\mathbf{r})|H|\psi(\mathbf{r}) \rangle = \langle \chi(\mathbf{p})|H|\chi(\mathbf{p}) \rangle$. Also, the Gaussian function is a smooth function and its derivative of any order is square integrable, thus any power of the Laplacian operator ∇^2 is a hermitian operator at least under Gaussian functions. Therefore, analyzing the Hamiltonian (13) with the variational method can be considered as reasonable even though solving the eigenvalue equation of the differential operator (13) may be confronted with the mathematical difficulties because of the square root operator in (13).

With the Gaussian trial wave function (11) or (14), the expectation value of the Hamiltonian (13) can be calculated easily besides the square root operator,

$$\langle \mathbf{p}^2 \rangle = \langle \psi(\mathbf{r})|\mathbf{p}^2|\psi(\mathbf{r}) \rangle = \langle \chi(\mathbf{p})|\mathbf{p}^2|\chi(\mathbf{p}) \rangle = \frac{3}{2}\mu^2, \tag{21}$$

$$\langle V(r) \rangle = \langle \psi(\mathbf{r})| -\frac{\alpha_c}{r} + Kr \, |\psi(\mathbf{r}) \rangle = \frac{2}{\sqrt{\pi}}(-\alpha_c \mu + K/\mu). \tag{22}$$

Now let us consider the expectation value of the square root operator in the momentum space

$$\langle \sqrt{\mathbf{p}^2 + m^2} \rangle = \langle \chi(\mathbf{p})|\sqrt{\mathbf{p}^2 + m^2}|\chi(\mathbf{p}) \rangle = \left(\frac{\mu}{\sqrt{\pi}}\right)^3 \int_0^\infty e^{-p^2/\mu^2} \sqrt{\mathbf{p}^2 + m^2} \, d^3p$$

$$= \frac{4\mu}{\sqrt{\pi}} \int_0^\infty e^{-x^2} \sqrt{x^2 + (m/\mu)^2} \, x^2 dx. \tag{23}$$

The integral (23) can be given as a series expansion by the following procedure. First, define

$$I(s) \equiv \int_0^\infty \sqrt{x^2 + s} \, x^2 e^{-x^2} dx = s^2 \int_0^\infty \sqrt{t^2 + 1} \, t^2 e^{-st^2} dt, \tag{24}$$

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$$I_0(s) \equiv \int_0^\infty \sqrt{x^2 + s}\, e^{-x^2} dx = s \int_0^\infty \sqrt{t^2 + 1}\, e^{-st^2} dt. \qquad (25)$$

Next, from (24) and (25), we find the following differential relations

$$\frac{d}{ds}\left(\frac{I_0}{s}\right) = -\frac{1}{s^2} I, \quad \frac{dI}{ds} = -\frac{1}{2} I_0 + I. \qquad (26)$$

Combining two equations in (26), we get a second order differential equation for $I(s)$,

$$s I''(s) - (1+s) I'(s) + \frac{1}{2} I(s) = 0. \qquad (27)$$

The series solution to (27) is given as

$$
\begin{aligned}
I(s) &= c_1 I_1(s) + c_2 I_2(s), \\
I_1(s) &= s^2 F(s; \tfrac{3}{2}, 3) = s^2 \left\{ 1 + \frac{1}{2}s + \frac{5}{32}s^2 + \frac{7}{192}s^3 + \frac{7}{1024}s^4 + \cdots \right\}, \quad (28) \\
I_2(s) &= I_1(s) \int \frac{s e^s}{[I_1(s)]^2} ds = -\frac{1}{16} s^2 \ln s \left(1 + \frac{1}{2}s + \frac{5}{32}s^2 + \cdots \right) \\
&\quad - \frac{1}{2} \left(1 + \frac{1}{2}s + \frac{5}{32}s^2 + \frac{7}{192}s^3 + \frac{7}{1536}s^4 + \cdots \right),
\end{aligned}
$$

where $F(s; \tfrac{3}{2}, 3)$ is the confluent hypergeometric function which is convergent for any finite s, and the integral constants $c_1 \simeq -0.095$, $c_2 = -1$. See Appendix for the derivation of these numerical values for c_i.

Finally, collecting (21), (22) and (23), the expectation value of H is written as

$$
\begin{aligned}
\langle H \rangle &= M + \frac{1}{2M}\left(\frac{3}{2}\mu^2\right) + \frac{2}{\sqrt{\pi}}(-\alpha_c \mu + K/\mu) \\
&\quad + \frac{2\mu}{\sqrt{\pi}}\left[1 + \frac{1}{2}(m/\mu)^2 + \left(\frac{5}{32} - 2c_1\right)(m/\mu)^4 + \frac{1}{4}(m/\mu)^4 \ln(m/\mu) \right], \quad (29)
\end{aligned}
$$

up to $(m/\mu)^4$.

With the input value of $m = m_{sp} = 0.15$ GeV, we minimize $\langle H \rangle$ of (29), and then we obtain

$$
\begin{aligned}
p_F = \bar{\mu} &= 0.54 \; GeV, \qquad m_B = \bar{E} = 5.54 \; GeV \qquad \text{for } \alpha_s = 0.35, \qquad (30) \\
\bar{\mu} &= 0.49 \; GeV, \qquad \qquad \quad \bar{E} = 5.63 \; GeV \qquad \text{for } \alpha_s = 0.24.
\end{aligned}
$$

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Here let us check how much sensitive our calculation of p_F is by considering the case where $m = m_{sp} = 0$ for comparison. For $m_{sp} = 0$ the integral in (23) is done easily and we obtain the following values of $\bar{\mu} = p_F$ by the above variational method.

$$\bar{\mu} \;=\; 0.53 \; GeV, \qquad \bar{E} = 5.52 \; GeV \qquad \text{for } \alpha_s = 0.35, \tag{31}$$

$$\bar{\mu} \;=\; 0.48 \; GeV, \qquad \bar{E} = 5.60 \; GeV \qquad \text{for } \alpha_s = 0.24.$$

As we see in (31), the results are similar to those in (30) where $m_{sp} = 0.15$ GeV. We could expect this insensitivity of the value of p_F to that of m_{sp} because the value of m_{sp}, which should be small in any case, can not affect the integral in (23) significantly.

The calculated values of the B-meson mass, $\bar{E}$, are much larger than the measured value of 5.28 GeV. The large values for the mass are originated partly because the Hamiltonian (29) does not take care of the correct spin dependences for B and B^*. The difference between the pseudoscalar meson and the vector meson is given arise to by the chromomagnetic hyperfine splitting, which is given by

$$V_s = \frac{2}{3Mm} \, \vec{s}_1 \cdot \vec{s}_2 \, \nabla^2(-\frac{\alpha_c}{r}). \tag{32}$$

Then the expectation values of V_s are given by

$$\langle V_s \rangle = -\frac{2}{\sqrt{\pi}} \frac{\alpha_c \mu^3}{Mm} \;\; \text{for} \;\; B, \quad \langle V_s \rangle = \frac{2}{3\sqrt{\pi}} \frac{\alpha_c \mu^3}{Mm} \;\; \text{for} \;\; B^*, \tag{33}$$

and we treat $\langle V_s \rangle$ only as a perturbation. Then with the input value of $m = m_{sp} = 0.15$ GeV, we get for B meson

$$p_F \;=\; 0.54 \; GeV, \qquad \bar{E}_B = 5.42 \; GeV \qquad \text{for } \alpha_s = 0.35, \tag{34}$$

$$p_F \;=\; 0.49 \; GeV, \qquad \bar{E}_B = 5.56 \; GeV \qquad \text{for } \alpha_s = 0.24,$$

and for B^*

$$p_F \;=\; 0.54 \; GeV, \qquad \bar{E}_{B^*} = 5.58 \; GeV \qquad \text{for } \alpha_s = 0.35, \tag{35}$$

$$p_F \;=\; 0.49 \; GeV, \qquad \bar{E}_{B^*} = 5.65 \; GeV \qquad \text{for } \alpha_s = 0.24.$$

The calculated values of the B-meson mass, 5.42 GeV ($\alpha_s = 0.35$) and 5.56 GeV ($\alpha_s = 0.24$) are in reasonable agreement compared to the experimental value of $m_B = 5.28$ GeV; the relative errors are 2.7% and 5.3%, respectively. However, for the Fermi momentum p_F, the calculated values, 0.54 GeV ($\alpha_s = 0.35$) and 0.49 GeV ($\alpha_s = 0.24$), are larger than the value 0.3 GeV, which has been commonly used in the experimental analyses of energy spectrum of semileptonic B-meson decay. The value $p_F = 0.3$ GeV corresponds to the B-meson radius $R_B \sim 0.66$ fm, which seems too large. On the other hand, the value $p_F = 0.5$ GeV corresponds to $R_B \sim 0.39$ fm, which looks in reasonable range.

4. Conclusion

The Gaussian width of Fermi motion, p_F, is the most important parameter of the ACCMM model, and the value $p_F \sim 0.3$ GeV has been commonly used in experimental analyses without clear theoretical or experimental evidence. Therefore, it is recommended to determine the value of p_F more reliably, when we think of its importance in experimental analyses. We calculated the value for p_F in the relativistic quark model using the variational method. We obtained $p_F = 0.54$ GeV, which is much larger than 0.3 GeV. We also derived the ground state eigenvalue of $E_B \simeq 5.5$ GeV, which is in reasonable agreement with the experimental value $m_B = 5.28$ GeV.

We studied the dependence of $|V_{ub}/V_{cb}|$ on the Fermi momentum parameter p_F in the ACCMM model, and extracted $|V_{ub}/V_{cb}|$ as a function of p_F. It shows that $|V_{ub}/V_{cb}|$ is very much dependent on the value of p_F. When we use $p_F = 0.5$ GeV instead of 0.3 GeV, $|V_{ub}/V_{cb}|$ is increased by a factor 1.81. Then the previous discrepancy between the ACCMM model and the Isgur $et\ al.$ model for the value of $|V_{ub}/V_{cb}|^2$ turns into a good agreement.

After we submitted this paper, we have been informed an interesting work by C. Greub *et. al.* [13], which reports a similar conclusion for the value of p_F, that is $p_F = 566$ MeV, even though they used totally different approach.

Appendix

The integration constants c_1 and c_2 in (28) are given by the following relations,

$$I(0) \quad = \quad -\frac{1}{2}c_2 = \int_0^\infty x^3 e^{-x^2} dx = \frac{1}{2}, \tag{36}$$

$$I''(s \approx 0) \quad = \quad 2c_1 + c_2(-\frac{1}{8}\ln s - \frac{11}{32})$$

$$= \quad -\frac{1}{4}\int_0^\infty x^2(x^2 + s)^{-3/2} e^{-x^2} dx \quad \text{at} \quad s \approx 0. \tag{37}$$

Then, from (36), we get

$$c_2 \quad = \quad -1. \tag{38}$$

The integral in (37) can be expanded as

$$J(s = a^2) \quad = \quad \int_0^\infty x^2(x^2 + a^2)^{-3/2} e^{-x^2} dx$$

$$= \quad \int_0^\infty x^2[(x + a)^2 - 2ax]^{-3/2} e^{-x^2} dx$$

$$= \quad \int_0^\infty x^2(x + a)^{-3}\left[1 - \frac{2ax}{(x + a)^2}\right]^{-3/2} e^{-x^2} dx$$

$$= \quad \sum_{n=0}^\infty \frac{(2n + 1)! a^n}{2^n (n!)^2} \int_0^\infty \frac{x^{n+2}}{(x + a)^{2n+3}} e^{-x^2} dx. \tag{39}$$

Next the integral in (39) is obtained by

$$\int_0^\infty \frac{x^{n+2}}{(x + a)^{2n+3}} e^{-x^2} dx = \frac{1}{(2n + 2)!}\left(\frac{\partial}{\partial a}\right)^{2n+2} \int_0^\infty \frac{x^{n+2}}{x + a} e^{-x^2} dx. \tag{40}$$

Again the integral in (40) is related to another integral

$$\int_0^\infty \frac{x^{n+2}}{x + a} e^{-x^2} dx = \sum_{k=0}^{n+1} \frac{(-a)^k}{2}\left(\frac{n - k}{2}\right)! + (-a)^{n+2} \int_0^\infty \frac{e^{-x^2}}{x + a} dx. \tag{41}$$

The integral in (41) is given, for an infinitesimal value of a, by integration by parts,

$$\int_0^\infty \frac{e^{-x^2}}{x + a} dx = e^{-x^2} \ln(x + a)|_0^\infty - \int_0^\infty e^{-x^2}(-2x)\ln(x + a)dx = -\ln a - \frac{\gamma}{2} + O(a), \tag{42}$$

where $\gamma \sim 0.5772$ is the Euler's constant. Collecting (40), (41), and (42),

$$J(a \approx 0) = \sum_{n=0}^\infty \frac{1}{2^n (n!)^2 (2n + 2)} a^n \left(\frac{\partial}{\partial a}\right)^{2n+2} (-a)^{n+2}\{-\ln a - \frac{\gamma}{2} + O(a)\}. \tag{43}$$

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To get the constant c_1, we should extract a logarithmic term and constants from (43),

$$J(a \approx 0) = (-ln\ a - \frac{\gamma}{2} - \frac{3}{2}) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2^n (n!)^2 (2n+2)} a^n \left(\frac{\partial}{\partial a}\right)^{2n+2} (a^{n+2} ln\ a), \quad (44)$$

$$
\begin{aligned}
a^n \left(\frac{\partial}{\partial a}\right)^{2n+2} (a^{n+2} ln\ a)|_{a=0} &= \sum_{k=0}^{n+2} \frac{(2n+2)!}{k!(2n+2-k)!} \frac{(n+2)!(2n+1-k)!(-1)^{k+1}}{(n+2-k)!} \\
&= (-1)^{n+1}(n+2)!(n-1)!. \quad (45)
\end{aligned}
$$

Inserting (45) into (44), we get

$$J(a \approx 0) = -ln\ a - \frac{\gamma}{2} - 1 + \beta, \quad (46)$$

where $\beta = \sum_{n=1}^{\infty} 1/(n2^n) \approx 0.6932$. Then, from (37), we get

$$c_1 = -\frac{3}{64} + \frac{\gamma}{16} - \frac{1}{8}\beta \approx -0.0975. \quad (47)$$

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Fig.1 The ratio $|V_{ub}(p_{_F})/V_{ub}(p_{_F} = 0.3)|$ as a function of $p_{_F}$.